ITCS 2175 HW 8 Answer Key

There are 130 points total over the entire homework.

Section 3.4, pp. 270-271, 12, 14
Section 7.1, pp. 480-482, 4, 30, 48e, 54

Section 3.3
12) [30 points]
Prove that $f_1^2 + f_2^2 + \ldots + f_n^2 = f_n \bullet f_{n+1}$ is true for all positive n where f_n is the n'th Fibonacci number.

Basis Step: We will show that the proposition is true for the least element P(1).
$f_1^2 = 1^2 = 1 \bullet 1 = f_1 \bullet f_2$.

Inductive Step: We want to show that P(n) → P(n+1).
Assume $f_1^2 + f_2^2 + \ldots + f_n^2 = f_n \bullet f_{n+1}$ is true for any n > 0.
We must show that $f_1^2 + f_2^2 + \ldots + f_{n+1}^2 = f_{n+1} \bullet f_{n+2}$ is then true.
$f_1^2 + f_2^2 + \ldots + f_{n+1}^2 = [f_1^2 + f_2^2 + \ldots + f_n^2] + f_{n+1}^2$
$= f_n \bullet f_{n+1} + f_{n+1}^2$ Using the inductive assumption
$= f_{n+1}(f_n + f_{n+1})$
$= f_{n+1} \bullet f_{n+2}$ Using the definition of Fibonacci numbers

This proves that $f_1^2 + f_2^2 + \ldots + f_n^2 = f_n \bullet f_{n+1}$ is true whenever n is a positive integer.

14) [30 points]
Prove that $f_{n+1} \bullet f_{n-1} - f_n^2 = (-1)^n$ whenever n is a positive integer where f_n is the n'th Fibonacci number.

Basis Step: We will show that P(1) is true.
$f_2 \bullet f_0 - f_1^2 = 1 \bullet 0 = (-1)^1$.

Inductive Step: We want to show that P(n) → P(n+1).
Assume $f_{n+1} \bullet f_{n-1} - f_n^2 = (-1)^n$ is true when n is a positive integer.
We must show that $f_{n+2} \bullet f_n - f_{n+1}^2 = (-1)^{n+1}$ is then true.
$f_{n+2} \bullet f_n - f_{n+1}^2 = (f_{n+1} + f_n)f_n - f_{n+1}^2$ Using the definition of f_{n+2}
$= f_n^2 + f_{n+1} \bullet f_n - f_{n+1}^2$
$= f_n^2 + f_{n+1}(f_n - f_{n+1})$
$= f_n^2 - f_{n+1}(f_{n+1} - f_n)$
$= f_n^2 - f_{n+1} \bullet f_{n-1}$ Using the definition of f_{n+1}
$= -1 \bullet (-1)^n$ Using the inductive assumption
$= (-1)^n + 1$

This proves that $f_{n+1} \bullet f_{n-1} - f_n^2 = (-1)^n$ whenever n is a positive integer.

Section 7.1

4) [5 points each]

a) Antisymmetric and transitive

b) Reflexive, symmetric, and transitive

c) Reflexive, symmetric, and transitive

d) Reflexive and symmetric.

30) [10 points]

$S \circ R = \{(1,1),(1,2),(2,1),(2,2)\}$

48e) [20 points]

Prove that $S \circ R$ is reflexive if R and S are reflexive relations on the set A.

Given that R and S are reflexive, $\forall a \in A$, $(a,a) \in R$ and $(a,a) \in S$. Then by the definition of the composite, $(a,a) \in S \circ R$, $\forall a \in A$. Thus $S \circ R$ is reflexive.

54) [5 points each]

Let R be the relation on the set {1,2,3,4,5} containing the ordered pairs:

{(1,1),(1,2),(1,3),(2,3),(2,4),(3,1),(3,4),(3,5),(4,2),(4,5),(5,1),(5,2),(5,4)}

a) $R^2 = R \circ R = \{(1,1),(1,2),(1,3),(1,4),(1,5),$
$(2,1),(2,2),(2,4),(2,5),$
$(3,1),(3,2),(3,3),(3,4),(3,5),$
$(4,1),(4,2),(4,3),(4,4),$
$(5,1),(5,2),(5,3),(5,4),(5,5)\}$

b) $R^3 = R^2 \circ R = \{(1,1),(1,2),(1,3),(1,4),(1,5),$
$(2,1),(2,2),(2,3),(2,4),(2,5),$
$(3,1),(3,2),(3,3),(3,4),(3,5),$
$(4,1),(4,2),(4,3),(4,4),(4,5),$
$(5,1),(5,2),(5,3),(5,4),(5,5)\} = A \times A.$

c) $R^4 = R^3 = A \times A.$

d) $R^5 = R^4 = A \times A.$