ITCS 2175 HW 4 Answer Key

There are 100 points total over the entire homework.
(These answers are just one possibility. There are other equivalent, correct answers.)

Section 1.6
1) [10 points]

a. $\{x \mid x = 3n \land n \in \mathbf{N} \land n \leq 4\}$
b. $\{x \mid x \in \mathbf{Z} \land -3 \leq x \leq 3\}$
c. { x | x is a letter of the English alphabet between m and p inclusive}

2) [8 points]

a. NO. Ø is a set with no elements and a power set has to have 2^n elements for $n \in \mathbf{N}$. Often people say YES because they confuse Ø with {Ø}, which are not equal. See Example 12 on page 82.
b. YES. {a}.
c. NO. This set has 3 elements, and a power set must have 2^n elements for $n \in \mathbf{N}$.
d. YES. {a,b}

3) [10 points]
Show that $A \times B \neq B \times A$ if A and B are non-empty and not equal.
Let A and B be non-empty sets. Thus $\exists a$ such that $a \in A$ and $\exists b$ such that $b \in B$. The Cartesian product $A \times B$ is the set of all ordered pairs (a,b), and the Cartesian product $B \times A$ is the set of all ordered pairs (b,a) where $a \in A$ and $b \in B$. But ordered pairs (a,b) and (c,d) are equal if and only if a = c and b = d (page 83). Thus (a,b) = (b,a) if and only if a = b. Thus $A \times B = B \times A$ if and only if all elements of A and B are equal (A = B).

Section 1.7
4) [10 points]
Let A = {a,b,c,d,e} and B = {a,b,c,d,e,f,g,h}

a. $A \cup B$ = {a,b,c,d,e,f,g,h}
b. $A \cap B$ = {a,b,c,d,e}
c. A – B = Ø
d. B – A = {f,g,h}

5) [10 points]
Find the sets A and B if A – B = {1,5,7,8}, B – A = {2,10}, and $A \cap B$ = {3,6,9}.
A = {1,3,5,6,7,8,9}
B = {2,3,6,9,10}

6)
Show the following
a. $(A \cup B) \subseteq (A \cup B \cup C)$ **[10 points]**

Let e be an arbitrary element of (A ∪ B). Then e∈A or e∈B or e is an element of both. It follows that e must then be an element of (A ∪ B ∪ C). Therefore (A ∪ B) ⊆ (A ∪ B ∪ C).

b. (A – B) – C ⊆ (A – C) **[10 points]**

Let e be and arbitrary element of (A – B) – C. If e ∈ (A – B) – C, then e ∈ (A – B) and e ∉ C. If e ∈ (A – B), then e ∈ A and e ∉ B. Thus e ∈ A and e ∉ C, which means e ∈ (A – C). Therefore (A – B) – C ⊆ (A – C).

c. (B – A) ∪ (C – A) = (B ∪ C) – A **[8 points for each part]**

i. Membership Table

A	B	C	B – A	C – A	**(B – A) ∪ (C – A)**	B ∪ C	**(B ∪ C) – A**
1	1	1	0	0	**0**	1	**0**
1	1	0	0	0	**0**	1	**0**
1	0	1	0	0	**0**	1	**0**
1	0	0	0	0	**0**	0	**0**
0	1	1	1	1	**1**	1	**1**
0	1	0	1	0	**1**	1	**1**
0	0	1	0	1	**1**	1	**1**
0	0	0	0	0	**0**	0	**0**

Since column (B – A) ∪ (C – A) is identical with column (B ∪ C) – A, they must be identical.

ii. Propositional Logic

(B – A) ∪ (C – A)

= {x | x ∈ (B – A) ∪ (C – A)}	Set builder notation
= {x | x ∈ (B – A) ∨ x ∈ (C – A)}	Def. of Union
= {x | (x ∈ B ∧ x ∉ A) ∨ (x ∈ C ∧ x ∉ A)}	Def. of Difference x2
= {x | (x ∉ A ∧ x ∈ B) ∨ (x ∉ A ∧ x ∈ C)}	Commutative x2
= {x | x ∉ A ∧ (x ∈ B ∨ x ∈ C)}	Distributive
= {x | (x ∈ B ∨ x ∈ C) ∧ x ∉ A }	Commutative
= {x | x ∈ (B ∪ C) ∧ x ∉ A }	Def. of Union
= {x | x ∈ ((B ∪ C) – A)}	Def. of Difference
= (B ∪ C) – A	Set builder notation

iii. Tabular proof using set identities

(B – A) ∪ (C – A)

$= (B \cap \overline{A}) \cup (C \cap \overline{A})$	Def. of Difference x2
$= (\overline{A} \cap B) \cup (\overline{A} \cap C)$	Commutative x2
$= \overline{A} \cap (B \cup C)$	Distributive
$= (B \cup C) \cap \overline{A}$	Commutative
$= (B \cup C) - A$	Def. of Difference

iv. Sentence-type logical argument

First we show that $(B - A) \cup (C - A) \subseteq (B \cup C) - A$.

Let e be an arbitrary element of $(B - A) \cup (C - A)$. Then either $e \in (B - A)$ or $e \in (C - A)$ or e is an element of both. If $e \in (B - A)$ then $e \in B$ and $e \notin A$. If $e \in (C - A)$ then $e \in C$ and $e \notin A$. In either case, $e \notin A$ and $e \in B$ or $e \in C$ or both. In other words, $e \in (B \cup C) - A$. Thus $(B - A) \cup (C - A) \subseteq (B \cup C) - A$.

Next we will show that $(B \cup C) - A \subseteq (B - A) \cup (C - A)$.

Let e be an arbitrary element of $(B \cup C) - A$. Then $e \notin A$ and $e \in (B \cup C)$. The latter means that $e \in B$ or $e \in C$ or e is an element of both. But this means that $e \in (B - A)$ or $e \in (C - A)$ or both. In other words, $e \in (B - A) \cup (C - A)$. Thus $(B \cup C) - A \subseteq (B - A) \cup (C - A)$.

Since $(B - A) \cup (C - A) \subseteq (B \cup C) - A$ and $(B \cup C) - A \subseteq (B - A) \cup (C - A)$, then $(B - A) \cup (C - A) = (B \cup C) - A$.