Sec. 1.2
In the following problems, use logical equivalences.

10. Show that each implication is a tautology.

a) $[\neg p \wedge (p \vee q)] \rightarrow q$

b) $[(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r)$

12. Determine whether **$(\neg p \wedge (p \rightarrow q)) \rightarrow \neg q$** is a tautology

16. Show that **$p \rightarrow q$** and **$\neg q \rightarrow \neg p$** are logically equivalent.

Sec. 1.3

12. Let F(x,y) be the statement "x can fool y," where the universe of discourse is the set of all people in the world. Use quantifiers to express each of the following statements:

c) Everybody can fool somebody.
d) There is no one who can fool everybody.

22. Determine the truth value of each of the following statements if the universe of discourse of each variable is the set of real numbers.

a) $\exists x(x^2 = 2)$
b) $\exists x(x^2 = -1)$
c) $\forall x \exists y(x^2 = y)$
d) $\forall x \exists y(x = y^2)$
e) $\exists x \forall y(xy = 0)$
f) $\exists x \exists y(x + y \neq y + x)$
g) $\forall x \neq 0 \exists y(xy = 1)$
h) $\exists x \forall y \neq 0(xy = 1)$
i) $\forall x \exists y(x + y = 1)$
j) $\exists x \exists y[(x + 2y = 2) \wedge (2x + 4y = 5)]$
k) $\forall x \exists y[(x + y = 2) \wedge (2x - y = 1)]$
l) $\forall x \forall y \exists z(z = (x + y)/2)$

24. Rewrite each of the following statements so that negations appear only within predicates (that is, so that no negation is outside a quantifier or an expression involving logical connectives). Use the transformations shown in Table 2. In other words, push all of the negation symbols inside the quantifiers, changing the sense of the quantifiers because of the equivalences in Table 2. You may also need to use DeMorgan's laws and double negation.

a) $\neg \exists x \exists y P(x,y)$
b) $\neg \forall x \exists y P(x,y)$
c) $\neg \exists y(Q(y) \wedge \forall x \neg R(x,y))$

d) $\neg\exists y(\exists x R(x,y) \vee \forall x S(x,y))$

e) $\neg\exists y(\forall x \exists z T(x,y,z) \vee \exists x \forall z U(x,y,z))$