Quiz 1
ITCS 2175

Full Name ______________________________

$p \wedge T \Leftrightarrow p$; $p \vee F \Leftrightarrow p$	Identity Laws
$p \vee T \Leftrightarrow T$; $p \wedge F \Leftrightarrow F$	Domination Laws
$p \vee p \Leftrightarrow p$; $p \wedge p \Leftrightarrow p$	Idempotent Laws
$\neg(\neg p) \Leftrightarrow p$	Double Negation Law
$p \vee q \Leftrightarrow q \vee p$; $p \wedge q \Leftrightarrow q \wedge p$	Commutative Laws
$(p \vee q) \vee r \Leftrightarrow p \vee (q \vee r)$ $(p \wedge q) \wedge r \Leftrightarrow p \wedge (q \wedge r)$	Associative Laws
$p \vee (q \wedge r) \Leftrightarrow (p \vee q) \wedge (p \vee r)$ $p \wedge (q \vee r) \Leftrightarrow (p \wedge q) \vee (p \wedge r)$	Distribution Laws
$\neg(p \vee q) \Leftrightarrow (\neg p \wedge \neg q)$ $\neg(p \wedge q) \Leftrightarrow (\neg p \vee \neg q)$	De Morgan's Laws
$p \vee \neg p \Leftrightarrow T$	Or Tautology
$p \wedge \neg p \Leftrightarrow F$	And Contradiction
$(p \rightarrow q) \Leftrightarrow (\neg p \vee q)$	Implication Equivalence

1. Prove that the product of two odd integers added to an odd integer is even. **(20 points)**

Proof: Let a and b be two odd integers. Then there exists integers m and n such that a = 2m+1 and b = 2n+1. Let c be an odd integer; then there exists and integer k such that c = 2k+1. ab + c = (2m+1)(2n+1) + 2k+1 = 4mn +2m +2n + 2k +2 = 2(2mn+m+n+k+1) which must be even since (2mn+m+n +k+1) is an integer.

Proof: Let a and b be two odd integers; c be an odd integer. **5pts**
Then there exists integers m and n such that a = 2m+1 and b = 2n+1; there exists an integer k such that c = 2k+1. **5pts**
ab + c = (2m+1)(2n+1) + 2k+1, etc… **5pts**
which must be even since (2mn+m+n+k+1) is an integer. **5pts**

2. Using the logical equivalencies listed on the front page of the quiz prove that $((p \vee q) \wedge \neg p) \rightarrow q$ is a tautology. **(20 points)**

$((p \vee q) \wedge \neg p) \rightarrow q$	$\Leftrightarrow$	$\neg((p \vee q) \wedge \neg p) \vee q$	Implication
	$\Leftrightarrow$	$(\neg(p \vee q) \vee \neg\neg p) \vee q$	DeMorgan
	$\Leftrightarrow$	$(\neg(p \vee q) \vee p) \vee q$	Double Neg.
	$\Leftrightarrow$	$\neg(p \vee q) \vee (p \vee q)$	Assoc.
	$\Leftrightarrow$	$(p \vee q) \vee \neg(p \vee q)$	Comm.
	$\Leftrightarrow$	T	Domination

3. Prove that $\sqrt{3}$ is irrational. **(20 points)**

Proof (by contradiction): Assume that $\sqrt{3}$ is rational, i.e. that $\sqrt{3} = a/b$ for $a,b \in N$ and $b \neq 0$. Since any fraction can be reduced until there are no common factors in the numerator and denominator, we can further assume that a and b have no common factors.

Then $3 = a^2/b^2$ which means that $3b^2 = a^2$. So a^2 is divisible by 3 (by the definition of divisibility). By the Lemma below, since a^2 is divisible by 3, a is also divisible by 3. Thus a = 3k and $3b^2 = 9k^2$. Thus $b^2 = 3k^2$ and b^2 is divisible by 3. But by the Lemma this means b is also divisible by 3. This means both a and b have a common factor (3), which is a contradiction of our original assumption. Therefore $\sqrt{3}$ is irrational.

Lemma: When m is a positive integer, then if m^2 is divisible by 3, then m is divisible by 3.
Proof: Use an indirect proof. If m is not divisible by 3, then m^2 is not divisible by 3. There are two cases to consider: m = 3k+1 and m = 3k+2 where $k \in N$.

Case 1
If m = 3k+1 then $m^2 = (3k+1)(3k+1) = 9k^2 + 6k + 1 = 3(3k^2 + 2k) + 1$ which is not divisible by 3.
Case 2
If m = 3k + 2 then $m^2 = (3k+2)(3k+2) = 9k^2 + 12k + 4 = 3(3k^2 + 4k + 1) + 1$ which is not divisible by 3.

4. Suppose that $P(x,y)$ is the statement $x+3y = xy$, where x and y are integers. What are the truth values of the following? (10 points)

T__________ $P(0,0)$

F__________ $\exists y P(3,y)$

F__________ $\exists x \forall y P(x,y)$

F_________ $\neg \forall x \neg \forall y P(x,y)$

T__________ $\exists x \exists y P(x,y)$

5. Use truth tables to prove or disprove that $\neg p \vee q \Leftrightarrow \neg p \rightarrow q$. **(10 points)**

p	q	¬p	¬p∨q	¬p→q
T	T	F	T	T
T	F	F	F	T
F	T	T	T	T
F	F	T	T	F

The two boxed columns are different, so the equivalence is not true.

6. Find the Disjunctive Normal Form of $(p \wedge q) \rightarrow (r \wedge q)$
(20 points)

p	q	r	$p \wedge q$	$(r \wedge q)$	$(p \wedge q) \rightarrow (r \wedge q)$
T	T	T	T	T	T
T	T	F	T	F	F
T	F	T	F	F	T
T	F	F	F	F	T
F	T	T	F	T	T
F	T	F	F	F	T
F	F	T	F	F	T
F	F	F	F	F	T

$(p \wedge q \wedge r) \vee (p \wedge \neg q \wedge r) \vee (p \wedge \neg q \wedge \neg r) \vee (\neg p \wedge q \wedge r) \vee (\neg p \wedge q \wedge \neg r) \vee (\neg p \wedge \neg q \wedge r) \vee$
$(\neg p \wedge \neg q \wedge \neg r)$